AN INTRODUCTION TO FLOW AND TRANSPORT IN FRACTAL MODELS OF POROUS MEDIA: PART I

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Abstract

This special issue gathers together a number of recent papers on fractal geometry and its applications to the modeling of flow and transport in porous media. The aim is to provide a systematic approach for analyzing the statics and dynamics of fluids in fractal porous media by means of theory, modeling and experimentation. The topics covered include lacunarity analyses of multifractal and natural grayscale patterns, random packing's of self-similar pore/particle size distributions, Darcian and non-Darcian hydraulic flows, diffusion within fractals, models for the permeability and thermal conductivity of fractal porous media and hydrophobicity and surface erosion properties of fractal structures.

Keywords: Flow and Transport; Fractal Model; Porous Media.

1. INTRODUCTION

Much attention has been paid to flow and transport phenomena in porous media in fields as diverse as petroleum engineering, soil science, engineering geology, chemical engineering, polymer composite manufacturing, and hydrology. Many natural and artificial porous media have been shown to follow fractal scaling rules, and a number of literature reviews[1–9] have appeared, from a range of theoretical and practical perspectives, since Mandelbrot[10] first introduced the theory of fractals and multifractals. It is no surprise that much attention and interest has been devoted to this topic over the past 20 years or so (Fig. 1). Figure 1 indicates increasing interest in both fractal geometry and flow and transport in porous media over time, with interest in flow and transport in porous media growing more rapidly than interest in fractals.

This special issue emphasizes fundamental scientific innovations related to the fluid flow and transport properties of porous media. It focuses on theoretical, numerical and experimental research, with an emphasis on contributions which increase the basic understanding of fractal theory applied to flow and transport through porous media, and its practical implications for engineering and science problems. It is apparent from the variety of papers included in this issue that fractal models can be extremely helpful for improving our understanding of the mechanisms of flow and transport in complex porous systems.

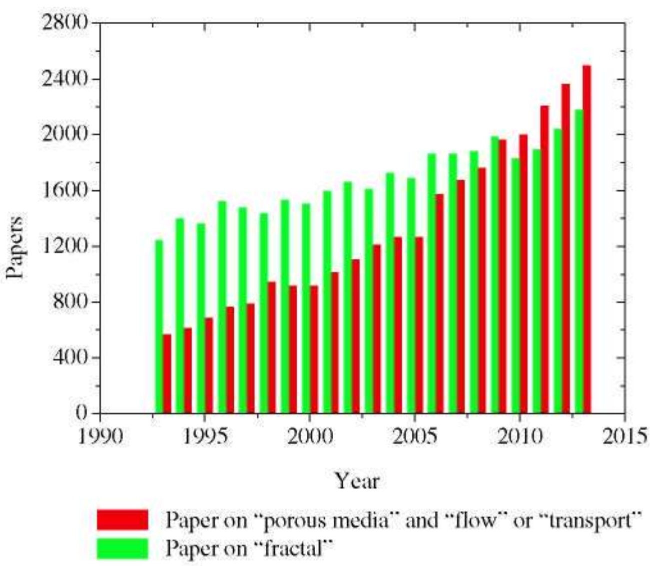

Fig. 1 The number of papers published over time related to flow/transport in porous media and fractals. The data are from the Web of Science$^{\mathrm{TM}}$ Core Collection (SCI-Expanded).

2. OVERVIEW OF WORK PRESENTED IN THIS SPECIAL ISSUE

The papers published in this issue of *Fractals* represent part I of a special topic covering fractal descriptions of flow and transport phenomena in porous media. They can be naturally divided into the following three groups.

The first group of papers focuses on fractal theory applied to pore- and particle-size distributions, which exert strong controls on flow and transport in earth materials. Roy and Perfect[11] have presented lacunarity analyses of multifractal and natural grayscale patterns. Lacunarity is a scale-dependent parameter that was developed for quantifying clustering in fractals and has subsequently been employed to characterize various natural patterns. These authors employed lacunarity analysis to discriminate between multifractal grayscale patterns with the same correlation dimension values, but different degrees of scale-dependent clustering. A new lacunarity parameter was formulated based on the weighted mean of the log-transformed lacunarity values at different scales. This parameter was then used to evaluate scale-dependent clustering in grayscale images of soil thin sections that had previously been classified as multifractals based on standard method of moments box-counting.

Martín *et al.*[12] have used two-dimensional (2D) computer simulation methods to generate granular porous media based on the random packing of particle size distributions that follow a self-similar, uniparametric model. The parameter p, which controls the model, is the proportion of mass of particles corresponding to the left half of the normalized size interval $[0, 1]$. It was shown that the parameter p and the fractal exponent of the associated mass power-law scaling relationship are efficient predictors of the packing behavior. Muñoz *et al.*[13] focus primarily on the scaling features of soil pore space as a 3D geometrical shape and on the ability of fractal dimensions to discriminate between two *a priori* different soil structures. The scaling features of pore size distribution, pore volume and pore-solid surface were investigated using X-ray computed tomography images of soil samples from two contrasting management practices at three different depths. Their results suggested that these fractal parameters could provide a comprehensive geometrical description of soil structure with fractal attributes of the soil pore space as a 3D shape. These geometrical descriptors should also probably include lacunarity to better understand and predict the complexity of soil functioning. Zhang *et al.*[14] presented a fractal pore structure model for shales. The effect of different pore structures on fractal dimension, and the influence of fractal dimension and pore size distribution on porosity were analyzed. Then they further developed a multilayer fractal adsorption model which takes into account the roughness of adsorption surface by using fractal theory.

The second group of papers focuses on diffusive and adjective flows in fractal porous media, and the intrinsic permeability and thermal conductivity of such media. By applying fractal theory to porous media, Liang *et al.*[15] were able to derive analytical expressions for the normalized average mass flux and pressure drop for power law fluid flow with pore wall effects considered. Their models were expressed as functions of power law index and structure parameters. On the basis of fractal scaling laws for the pore-size distribution, Xu *et al.*[16] developed a probability model for the thermal conductivity of porous media. They combined fractal geometry with the Monte Carlo technique. The resulting model indicates that the macroscopic thermal conductivity of the porous medium is a function of the thermal conductivities of the micro-structural constituents. Wu *et al.*[17] combined Wu-fractal/Ergun high-speed non-Darcy flow theory and dynamic system instability analysis to reveal the major influence factors of mine water inrush.

Wang *et al.*[18] derived a permeability–pore relationship for porous media, and theoretically described the interrelationships among the permeability, pore fractal dimension, porosity and pore structure parameters. An algorithm to generate 2D pore spaces with arbitrary fractal dimension was developed. Using the series–parallel flow resistance model and lattice Boltzmann method implemented in a 2D context, the effects of physical properties on the fluid flows were systematically analyzed in their work. Zheng *et al.*[19] constructed three kinds of porous media with the same porosity, different pore size distributions and fractal spectral dimensions using the random growth method. The different media were used to theoretically study the impacts of microscopic pore structures on water vapor diffusion processes in porous media.

The third group of papers focuses on the hydrophobicity and surface erosion properties of fractal structures. Quan *et al.*[20] studied fractal aspects of superhydrophobic surface coatings and their roles in delaying condensate and frost formation. The rough surfaces of copper foils, obtained by the solution immersion method, were shown to exhibit fractal scaling. The hydrophobicity of copper surfaces was enhanced with these fractal structures. The authors investigated the relationship between contact angles and fractal dimensions for surface roughness of the Cu samples obtained with different etching times. Xu *et al.*[21] used fractal geometry to describe the aggregate structure within cohesive sediments, and proposed a fractal model for the erosion threshold and the rate of surface erosion, which were expressed as functions of the median diameter and excess density.

3. CONCLUSIONS

The use of fractal geometry to characterize and model flow and transport in natural and engineered porous media is an extremely active field of research in many different disciplines.[22] As a result, it has been possible to extend classical expressions, such as the Lucas–Washburn equation, to describe heterogeneous systems by introducing fractal concepts.[23,24] Papers in the present volume have summarized existing theory and presented some new challenges for fractal geometry and its applications to flow and transport in porous materials. The guest editors hope that they will help to further advance this multidisciplinary endeavor.

ACKNOWLEDGMENTS

The guest editors would like to express their gratitude to all of the authors for their inspiring contributions and the anonymous referees for their efforts in providing valuable comments and feedback, which led to the high quality of this special issue. The first guest editor, J. C. Cai, would like to acknowledge Prof. E. Perfect for his great help and cooperation while visiting his group in Knoxville, TN, and the National Natural Science Foundation of China (41102080) and the Fundamental Research

Funds for the Central Universities (CUG130404 and CUG130103) for supporting his series of studies on flow and transport in fractal porous media. The guest editors M. A. Martín and F. San José Martínez would like to acknowledge Spain's Plan Nacional de Investigación Científica, Desarrollo e Innovación Tecnológica (I+D+I) (AGL2011-25175) for funding their research on this topic.

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